Dispersive Modes in the Time Domain: Analysis and Time-Frequency Representation

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Abstract— **Four algorithms for time-frequency (TF) distributions are considered for the processing and interpretation of dispersive time-domain (TD) data: The short-time Fourier transform, frequency and time-domain wavelets, and a new ARMA-based representation. The TF resolutions of the various distributions are discussed and compared with reference to results for the scattered fields from a chirped finite grating excited by a pulsed plane wave. The processing in the TF phase space extracts TD phenomenology, in particular the instantaneous dispersion relation—with its associated time-dependent frequencies—descriptive of the local TD Floquet modes on the chirped truncated grating.**

WITH the trend toward wideband (WB) transient waveforms, it is important to understand highly dispersive structure-induced wave phenomena directly in the time domain (TD), because the frequency-dependent scattering angles of the WB waveforms vary drastically over the bandwidth. Examples are provided by leaky modes (LM) on layered configurations [1], [2] and by Floquet modes (FM) excited by gratings [3], [4]. In a comprehensive study based on the high-frequency asymptotic behavior of rigorously formulated TD scattered fields, we have identified novel TD LM and FM which, although relating specifically to the layered [1], [2] and grating [3], [4] configurations, describe TD phenomena due to structure-induced dispersion in general. The asymptotics, due to inherent localization [5], [6], parametrize the wave physics in terms of compact wave objects which can be forward and backward propagated for wave-oriented data processing in the (space-time)-(wavenumber-frequency) phase space. (For space-wavenumber processing, see [7].) The accuracy of the algorithms has been verified in [1]–[4].

In this letter, we concentrate on the TD characteristics of FM observed at a fixed location as a function of time. The scattered TD signal is synthesized by inverting a frequency-domain FM Fourier integral asymptotically [3], [4], and yields a result parametrized by one or more time-dependent frequencies localized around stationary points. Because the FM are dispersive, the instantaneous frequencies place time-dependent constraints on the modal wavenumbers through the FM dispersion relation. Thus, a TD dispersive mode is a wavetrain with varying oscillation frequency dictated by its instantaneous dispersion inside an envelope weighted by the frequency spectrum of the input pulse [3],[4]. Specifically, we consider an N-element weakly aperiodic grating of wires parallel to the y-axis on the $z = 0$ plane and located along the x-axis at points x_n, for $n = 0, 1, 2, \ldots, N-1$. For weak departure from strict periodicity, we have shown [4] that, by introducing the function $x(v)$ through the replacement of the discrete index$n$ by the continuous variable $v(n \to v)$, we can define a local period $d(v) = dx(v)/dv \equiv x'(v)$ in the vicinity of $x(v)$. We have also shown that the instantaneous frequency associated with a $m \neq 0$ TD-FM excited by a normally incident plane wave is [4]

$$\omega_m(x,z,t) = \pm\frac{c\tau 2\pi|m|}{d_m(t)\sqrt{\tau^2 - z^2}},$$
$$d_m(t) = z'[v_m(t)], m = \pm 1, \pm 2, \cdots. \quad (1)$$

Here, $\tau = ct$ with c the speed of light in vacuum,m tags the various FM that propagate away from the grating, and $x[v_m(t)]$ identifies the localized region on the grating aperture from where the rays of the mth FM, which reach the observer at (x, z) at time t, originate. The instantaneous dispersion relation [4] $\sin\theta_m(t) = m\lambda_m(t)/d_m(t)(\lambda =$ wavelength) reveals that all $m \neq 0$ FM travel to the observer at the same time-dependent angle $\theta_m(t) = \sin^{-1}[1 - (z/\tau)^2]^{1/2}$. This allows synthesis of highly resolved short-pulse (SP) scatterings from the collection of individual wires by superposition, at each instant of time, of the various interfering TD-FM wavetrains with their distinct frequencies given by (1). The analysis above has been generalized to the case of an obliquely incident-pulsed plane wave [8]; the wave physics in that case is similar to that for normal incidence but with an escalation in algebraic complexity.

A useful way to display the role of time-dependent localized frequencies in scattering data is via time-frequency (TF) phase-space representations, as examined recently by several authors. Attention has been given to the Wigner transform [9], the short-time Fourier transform (STFT) [9]–[11], and a frequency-domain wavelet transform [10], [11]. Here we look at the time-dependent frequencies and instantaneous dispersion relations by comparing four different TF processing schemes for a grating example, which comprises $20 (N = 20)$ wire elements at $x_n = d_o(n + n^2\alpha/2)$; the aperiodicity parameter is $\alpha = .0025$, corresponding to a maximum variation of nearly 5% in the local period over the extent of the aperiodic grating.

For the problem conditions listed in its caption, Fig. 1 exhibits results for the TD scattered field (bottom plot), for its conventional global frequency spectrum (left-side plot),

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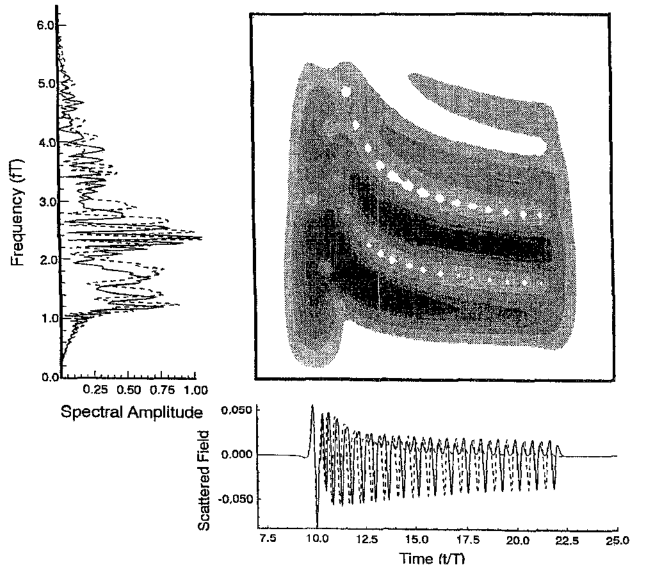

Fig. 1. Scattering data and processing (see text) for a 20 ($N = 20$) element grating of thin wires parallel to the$y$-axis at $z = 0$ and located along the x-axis at $x_n = d_o(n + n^2\alpha/2)$ for $n = 0, 1, 2, \ldots, N-1$; the aperiodicity parameter is $\alpha = .0025$. The normally incident-pulsed plane wave is described by a Raleigh wavelet [2, 3, 12] with center wavelength $\lambda_c = d_o/2$, and the scattered field is observed at a distance of $20\lambda_c$ directly above the right-most wire. Time is normalized to $T = d_o/c$, where c is the speed of light in vacuum. The TD scattered field and the global frequency spectrum for the same finite unchirped grating ($\alpha = 0$) are shown dashed.

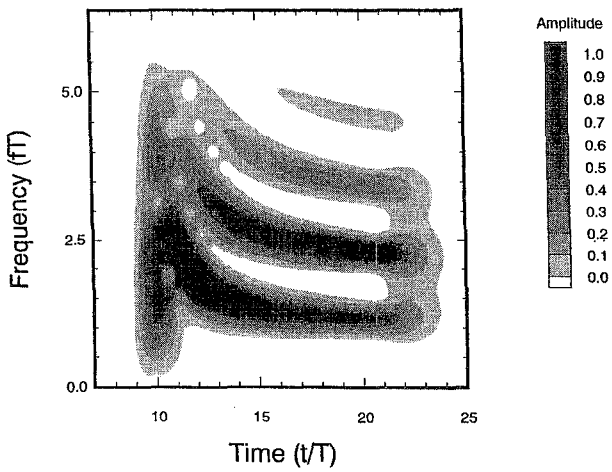

Fig. 2. FD wavelet transform [10], [11] of scattering data in Fig. 1 from aperiodic grating. The FD wavelet is adjusted such that in the early time the STFT Gaussian window is narrow and provides high temporal resolution; as the window is moved to the late time, it widens and provides good frequency (poor temporal) resolution. The standard deviation of the Gaussian window is σ =0.481T at 9.872T and grows to 1.333T at the end of TD waveform

and for its instantaneous spectrum obtained via STFT with a Gaussian window having a standard deviation $\alpha = 0.602T$ (center plot). Also shown is the scattered field for the same finite unchirped grating ($\alpha = 0$). One observes that even a weak chirp may introduce significant departures from the periodic case. The results demonstrate nicely how the STFT parametrizes the data in terms of instantaneous physical wave processes that are hidden completely in the global FT. One discerns two distinct TF signatures: Those with short-time duration, broad instantaneous bandwidth and negligible dispersion (vertical bands) at the beginning and the end, and those with long-time duration, narrow instantaneous bandwidth and strong dispersion (curved bands). The former represent SP wavefronts which sweep past the observer during a time interval equal to the pulse duration whereas the latter represent sustained oscillatory waves associated with TD-FM [3]. Scattering is seen to occur only over a finite time interval, implying a scatterer of finite extent, with weak body resonances. Taken together, the STFT features characterize clearly the scattering from a finite aperture whose scattering-induced equivalent excitation is the superposition of several distinct dispersive wavefields; the initial and final nondispersive pulsed events are due to diffraction at the truncations and (or) due to a nondispersive degenerate mode. This interpretation, inferred directly from the STFT, is in complete accord with the analytic model [4].

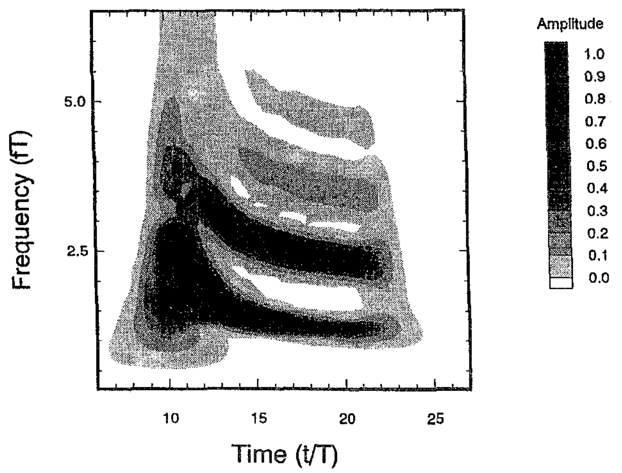

Fig. 3. Time domain Morlet [13] wavelet transform of scattering data in Fig. 1 corresponding to the aperiodic finite grating. The wavelet has 2.5 periods of a sine wave within the 3db point of the Gaussian envelope; the Gaussian envelope has a standard deviation of σ =2.15T at frequency 0.69fT and a standard deviation of σ =0.32T at frequency 5.67fT.

The STFT, which sorts out the basic physics, has constant TF resolution. To home in better on the modal dispersion relation (in our case, the TF curve from (1)), we consider variable-width windowing via two versions of the wavelet transform. The first, implemented in the frequency domain (FD), was developed by Ling and Kim [10], [11] and is essentially a variable window STFT. The FD wavelet (in our case, a modulated Gaussian) is described in the caption of Fig. 2. Compared with the STFT (Fig. 1), the results in Fig. 2 do show narrowed definition of the instantaneous dispersion bands. However, this wavelet transform is known to require data for which the early and late time response are discernable clearly. In Fig. 3 the TD transform using the Morlet "mother wavelet" [13] also implements the high (poor) temporal-poor (high) frequency resolution tradeoff but without the restrictive a priori discrimination between early and late times. Since the TD wavelet transform gathers low- and high- frequency information by using wide and narrow time windows, respectively, the temporal resolution at low frequencies is poor (see the early-time nondispersive return) but improves at higher frequencies with correspondingly poorer frequency resolution

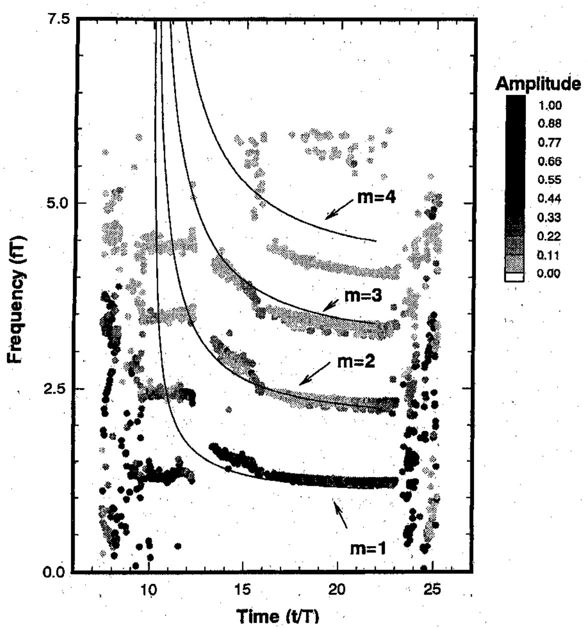

Fig. 4. ARMA processing of scattering data in Fig. 1 corresponding to the aperiodic finite grating. ARMA processing is performed to extract resonant frequencies and their residues over windowed portions of the time-domain data. A sliding Gaussian window is used as in Fig. 1, with a standard deviation of σ =0.602T. For each window position, the poles associated with models ARMA (8, 1) through ARMA (12, 12) are calculated along with their residues. Each dot in this figure represents a pole from one of the ARMA models and its intensity is indicated by the grey scale. The curves represent the time-dependent dispersion curves of the TDFM as given by (1).

(note that the vertical spread of the modal TF bands is wider at higher frequencies). The final TF processing scheme is based on a newly developed Auto-Regressive Moving Average (ARMA) algorithm [14] that guarantees stable spectral pole (and residue) extraction within a sliding TD Gaussian window (as was used in the STFT). The poles, weighted by their respective residues, yield the TF tracks in Fig. 4, which are seen to coincide closely with the instantaneous frequencies predicted in (1). The dispersion curves of the $m = 1, 2$, and 3 TDFM modes are predicted very accurately with the ARMA scheme, while the predicted $m = 4$ curve is slightly lower than that given by (1). This may be attributed to weak excitation of the $m = 4$ mode since its time-dependent frequencies are at the upper edges of the frequency spectrum (see Fig. 1). An additional advantage of the ARMA scheme is its excellent stability even in the presence of noise.

In conclusion, we have processed numerical scattering data for a chirped truncated grating excited by a pulsed plane wave to demonstrate extraction of information pertaining to TD dispersive phenomenology. Four different TF processings have been presented and compared. The STFT, which is least beset by artifacts, provides a good first cut at the phenomenology of TD wave objects in the TF phase space. The wavelet transforms selectively resolve the STFT bands around the dispersion curves, tracing out instantaneous frequencies. For the example here, the windowed-ARMA scheme provides the best TF resolution.

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